

Your homeowners policy would cover it in the majority of cases. The main exception would be if your neighbor was negligent in removing a dead tree

and you had warned him that it was dead and needed to be removed. In that case your neighbor's homeowners liability insurance should cover the loss.